

MARKSCHEME

November 2002

MATHEMATICS

Higher Level

Paper 1

Paper 1 Markscheme

Instructions to Examiners

Note: The number of marks for each question has been increased to 6. Where there are 2 marks (e.g. M2, A2) for an answer do NOT split the marks unless otherwise instructed.

1 Method of Marking

- (a) All marking must be done using a **red** pen.
- (b) In this paper, the maximum mark is awarded for a **correct answer**, irrespective of the method used. Thus, if the correct answer appears in the answer box, award the maximum mark and move onto the next question; in this case there is no need to check the method.
- (c) If an **answer is wrong**, then marks should be awarded for the method according to the markscheme. (A correct answer incorrectly transferred to the answer box is awarded the maximum mark.)

2 Abbreviations

The markscheme may make use of the following abbreviations:

- M** Marks awarded for **Method**
- A** Marks awarded for an **Answer** or for **Accuracy**
- G** Marks awarded for correct solutions, generally obtained from a **Graphic Display Calculator**, irrespective of working shown
- C** Marks awarded for **Correct** answers (irrespective of working shown)
- R** Marks awarded for clear **Reasoning**

3 Follow Through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

- (i) penalise the error when it **first occurs**;
- (ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent working;
- (iii) award **M** marks for a correct method and **A(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The errors made by a candidate may be: arithmetical errors; errors in algebraic manipulation; errors in geometrical representation; use of an incorrect formula; errors in conceptual understanding.

The following illustrates a use of the **follow through** procedure.

Markscheme		Candidate's Script	Marking
$\$ 600 \times 1.02$	<i>MI</i>	Amount earned = $\$ 600 \times 1.02$	✓ <i>MI</i>
= $\$ 612$	<i>AI</i>	= $\$ 602$	× <i>A0</i>
$\$ (306 \times 1.02) + (306 \times 1.04)$	<i>MI</i>	Amount = $301 \times 1.02 + 301 \times 1.04$	✓ <i>MI</i>
= $\$ 630.36$	<i>AI</i>	= $\$ 620.06$	✓ <i>AI(ft)</i>

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

- (i) **fewer** marks should be awarded at the discretion of the Examiner;
- (ii) marks awarded should be followed by '(d)' (to indicate that these marks have been awarded at the **discretion** of the Examiner);
- (iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

- (a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

- (i) a mark should be awarded followed by '(d)' (to indicate that the marks have been awarded at the **discretion** of the Examiner);
- (ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

Where alternative methods for complete questions are included, they are indicated by **METHOD 1, METHOD 2, etc.** Other alternative solutions, including graphic display calculator alternative solutions are indicated by **OR**. For example:

$$\begin{aligned} \text{Mean} &= 7906/134 && \text{(M1)} \\ &= 59 && \text{(A1)} \end{aligned}$$

OR

$$\text{Mean} = 59 \qquad \qquad \qquad \text{(G2)}$$

- (b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$. These equivalent numerical or algebraic forms may be written in brackets after the required answer.
- (c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7, 1·7, 1,7; different forms of vector notation such as \vec{u} , \bar{u} , \underline{u} ; $\tan^{-1} x$ for $\arctan x$.

5 Accuracy of Answers

There are two types of accuracy errors, incorrect level of accuracy, and rounding errors. Unless the level of accuracy is specified in the question candidates should be penalized **once only IN THE PAPER** for any accuracy error (**AP**). This could be an incorrect level of accuracy, or a rounding error. Hence, on the **first** occasion in the paper when a correct answer is given to the wrong degree of accuracy, or rounded incorrectly, maximum marks are **not** awarded, but on **all subsequent occasions** when accuracy errors occur, then maximum marks **are** awarded.

There are also situations (particularly in some of the options) where giving an answer to more than 3 significant figures is acceptable. This will be noted in the markscheme.

(a) Level of accuracy

- (i) In the case when the accuracy of the answer is **specified in the question** (for example: “find the size of angle A to the nearest degree”) the maximum mark is awarded **only if** the correct answer is given to the accuracy required.
- (ii) When the accuracy is **not** specified in the question, then the general rule applies:

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

(b) Rounding errors

Rounding errors should only be penalized at the **final answer** stage. This does **not** apply to intermediate answers, only those asked for as part of a question. Premature rounding which leads to incorrect answers should only be penalized at the answer stage.

Incorrect answers are wrong, and should not be considered under (a) or (b).

Examples

A question leads to the answer 4.6789....

- 4.68 is the correct 3 s.f. answer.
- 4.7, 4.679 are to the wrong level of accuracy, and should be penalised the first time this type of error occurs.
- 4.67 is incorrectly rounded – penalise on the first occurrence.

Note: All these “incorrect” answers may be assumed to come from 4.6789..., even if that value is not seen, but previous correct working is shown. However, 4.60 is wrong, as is 4.5, 4.8, and these should be penalised as being incorrect answers, not as examples of accuracy errors.

Markscheme		Candidate's Script (A)	Marking
(a) $a = 2.31 \times 3.43$ $= 7.9233 = 7.92$ (3 s.f.)	<i>MI</i> <i>AI</i>	(a) $a = 2.31 \times 3.43$ $= 7.9233 = 7.92$	<i>MI</i> <i>AI</i>
(b) $2a = 2 \times 7.9233$ $= 15.8466 = 15.8$ (3 s.f.)	<i>AI</i> <i>AI</i>	(b) $2a = 2 \times 7.29 = 14.58$ $= 14.5$	<i>A0</i> <i>A0(AP)</i>
		Total	<i>2 marks</i>

Notes: Award *AI* for either the exact answer 7.9233 or the 3 s.f. answer 7.92.

In line 3, Candidate A has incorrectly transcribed the answer for part (a), but then performs the calculation correctly, and would normally gain the follow through marks. However, the final answer is incorrectly rounded, and the **AP** applies.

Candidate's Script (B)	Marking	Candidate's Script (C)	Marking
(a) $a = 2.31 \times 3.43 = 7.9233$ $= 7.92$	<i>MI</i> <i>AI</i>	(a) $a = 2.31 \times 3.43 = 7.9233$ $= 7.93$	<i>MI</i> <i>A0(AP)</i>
(b) $2a = 2 \times 7.9233$ $= 15.8466 = 15.85$	<i>AI</i> <i>A0(AP)</i>	(b) $2a = 2 \times 7.93$ $= 15.86 = 15.8$	<i>AI(ft)</i> <i>AI(ft)</i>
Total	<i>3 marks</i>	Total	<i>3 marks</i>

Notes: Candidate B has given the answer to part (b) to the wrong level of accuracy, **AP** applies.

Candidate C has incorrectly rounded the answers to both parts (a) and (b), is penalised (**AP**) on the first occurrence (line 2), and awarded follow through marks for part (b).

Candidate's Script (D)	Marking	Candidate's Script (E)	Marking
(a) $a = 2.31 \times 3.43$ $= 7.923 = 7.9$	<i>MI</i> <i>A0(AP)</i>	(a) $a = 2.31 \times 3.43 = 7.923$ $= 7.93$	<i>MI</i> <i>A0(AP)</i>
(b) $2a = 2 \times 7.923$ $= 19.446 = 19.5$	<i>AI(ft)</i> <i>A0</i>	(b) $2a = 2 \times 7.93$ $= 15.86$	<i>AI(ft)</i> <i>AI(ft)</i>
Total	<i>2 marks</i>	Total	<i>3 marks</i>

Notes: Candidate D has given the answer to part (a) to the wrong level of accuracy, and therefore loses 1 mark (**AP**). The answer to part (b) is wrong.

Candidate E has incorrectly rounded the answer to part (a), therefore loses 1 mark (**AP**), is awarded follow through marks for part (b), and does **not** lose a mark for the wrong level of accuracy.

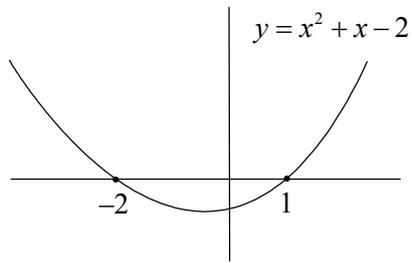
6 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks.

1. $f(x) = x^4 + ax + 3$
 $f(1) = 8$
 $1 + a + 3 = 8$
 $a = 4$
- (M1)(A1)
(M1)(A1)
(A2) (C6)
[6 marks]
2. $g(x) = f(x-1) - 1$
 $= 2(x-1)^3 - 3(x-1)^2 + (x-1) + 1 - 1$
 $= 2x^3 - 9x^2 + 13x - 6$
- (M1)(M1)
(M1)(A1)
(A1)(A1) (C6)
[6 marks]
3. The coefficient of x^3 is $\binom{8}{3} \left(\frac{-1}{2}\right)^3$
The coefficient of x^3 is -7
- (M2)(A2)
(A2) (C6)
[6 marks]
4. $y = 1 - \frac{8}{x^2 - 5x + 4}$
 $= 1 - \frac{8}{(x-4)(x-1)}$
Asymptotes are $y = 1$,
 $x = 4, x = 1$.
- (M1)(A1)
(A1)
(A1) (C2)
(A1)(A1) (C2)(C2)
[6 marks]
5. (a) The number of multiples of 4 is 250.
Required probability = 0.25.
- (M1)
(A1) (C2)
- (b) The number of multiples of 4 and 6 is
the number of multiples of 12
= 83.
Required probability = 0.083
- (M1)
(A1)
(A1)
(A1) (C4)
[6 marks]
6. $\sum_{r=1}^{50} \ln(2^r) = \sum_{r=1}^{50} r(\ln 2)$
 $= (\ln 2) \sum_{r=1}^{50} r$
 $= (\ln 2) \left(\left(\frac{50}{2} \right) 51 \right)$
 $= 1275 \ln 2$
- (M1)
(M1)(A1)
(M1)(A1)
(A1) (C6)
[6 marks]

7. (a)

METHOD 1



$$f(g(x)) = f(x^2 + x) = \sqrt{x^2 + x - 2}$$

$$\Rightarrow x^2 + x - 2 \geq 0$$

$$\Rightarrow x \leq -2, x \geq 1$$

$$a = -2, b = 1$$

(M1)(A1)

(M1)

(A1)(A1)

(C5)

METHOD 2

$$f(g(x)) = \sqrt{x^2 + x - 2}$$

$$= \sqrt{(x+2)(x-1)}$$

$$\Rightarrow (x+2)(x-1) \geq 0$$

$$\Rightarrow a = -2, b = 1$$

(M1)(A1)

(M1)

(A1)(A1)

(C5)

(b) range is $y \geq 0$

(A1)

(C1)

[6 marks]

8. $\bar{x} = \frac{2+3+6+9+a+b}{6}$

(M1)

$$= \frac{20+a+b}{6} = 6$$

$$\Rightarrow a+b=16$$

(A1)

$$\text{variance} = \frac{\sum_{i=1}^6 (x_i - 6)^2}{6}$$

$$= \frac{4^2 + 3^2 + 0^2 + 3^2 + (a-6)^2 + (b-6)^2}{6} = 10$$

(M1)

$$\Rightarrow (a-6)^2 + (b-6)^2 = 26$$

$$\Rightarrow (a-6)^2 + (10-a)^2 = 26$$

(M1)

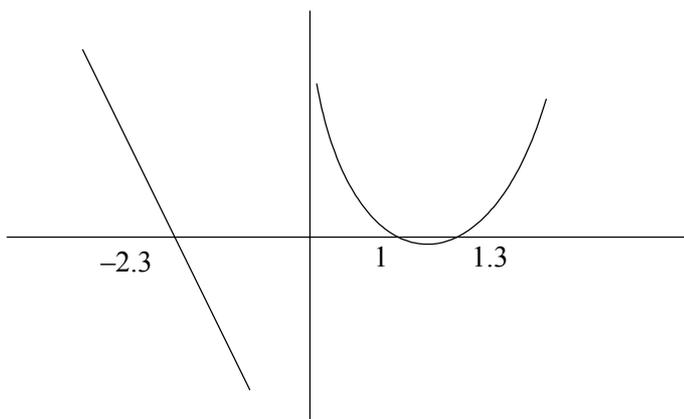
Therefore $a = 5, b = 11$

(A1)(A1)

(C6)

[6 marks]

9. METHOD 1



(M2)

$$x^2 - 4 + \frac{3}{x} < 0$$

$$\Rightarrow -2.30 < x < 0 \text{ or } 1 < x < 1.30$$

(G2)(G2)

(C6)

METHOD 2

$$x^2 - 4 + \frac{3}{x} < 0$$

$$\Rightarrow \frac{x^3 - 4x + 3}{x} < 0$$

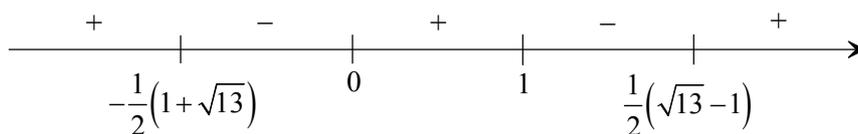
(M1)

$$\Rightarrow \frac{(x-1)(x^2+x-3)}{x} < 0$$

(M1)

$$\text{Critical values: } 1, \frac{1}{2}(-1 \pm \sqrt{13}), 0$$

(A2)



$$\Rightarrow -\frac{1}{2}(\sqrt{13}+1) < x < 0 \text{ or } 1 < x < \frac{1}{2}(\sqrt{13}-1)$$

(A1)(A1)

(C6)

[6 marks]

10. METHOD 1

$$\begin{array}{rcccc}
 R_1: & 1 & 2 & -3 & 2 \\
 R_2: & 2 & 3 & -5 & 3 \\
 \hline
 R_1: & 1 & 2 & -3 & 2 \\
 R_2 - 2R_1: & 0 & -1 & 1 & -1
 \end{array}
 \qquad (M2)(A1)$$

Let $z = t$, then $y = t + 1$ and $x = t$.

Therefore the line of intersection is $x = t, y = t + 1, z = t$ (or equivalent). (A1)(A1)(A1) (C6)

METHOD 2

$$\begin{aligned}
 \text{Let } z = 0 &\Rightarrow x + 2y = 2 && (M1) \\
 &2x + 3y = 3 && (A1) \\
 &\Rightarrow x = 0, y = 1
 \end{aligned}$$

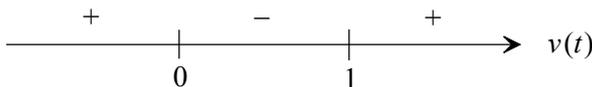
The direction vector of the line of intersection is

$$\begin{aligned}
 &\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -3 \\ 2 & 3 & -5 \end{vmatrix} && (M1) \\
 &= -\mathbf{i} - \mathbf{j} - \mathbf{k} && (A1)
 \end{aligned}$$

Therefore the line of intersection is $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ (or equivalent) (A2) (C6)

[6 marks]

11. $v(t) = 6t^2 - 6t = 6t(t - 1)$



METHOD 1

$$\begin{aligned}
 \text{Distance travelled} &= -\int_0^1 (6t^2 - 6t) dt + \int_1^2 (6t^2 - 6t) dt && (M1)(M1) \\
 &= 1 + 5 && (G1)(G1) \\
 &= 6 \text{ m.} && (G2) (C6)
 \end{aligned}$$

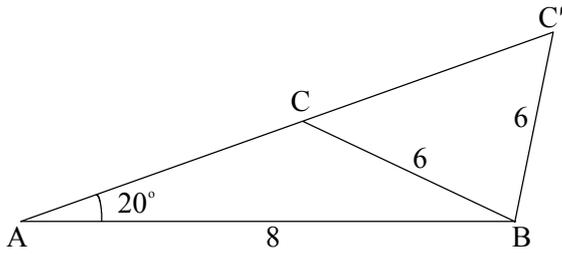
METHOD 2

$$\begin{aligned}
 \text{Distance travelled} &= -\int_0^1 (6t^2 - 6t) dt + \int_1^2 (6t^2 - 6t) dt && (M1)(M1) \\
 &= -[2t^3 - 3t^2]_0^1 + [2t^3 - 3t^2]_1^2 && (A1)(A1) \\
 &= -(-1) + 2(7) - 3(3) \\
 &= 6 \text{ m.} && (A2) (C6)
 \end{aligned}$$

Note: Award (G1)(or (A1)) if the units are missing.

[6 marks]

12. METHOD 1



$$\frac{\sin C}{8} = \frac{\sin 20}{6} \Rightarrow \sin C = 0.4560$$

(M1)(A1)

(From diagram) smallest triangle when \hat{C} is obtuse,
i.e. $\hat{C} = 152.9^\circ \Rightarrow \hat{CBA} = 7.13^\circ$ (or 7.1°)

(A1)(A1)

$$\text{Area } \triangle ABC = \frac{1}{2}(8)(6)(\sin 7.13^\circ) \text{ (or } \sin 7.1^\circ)$$

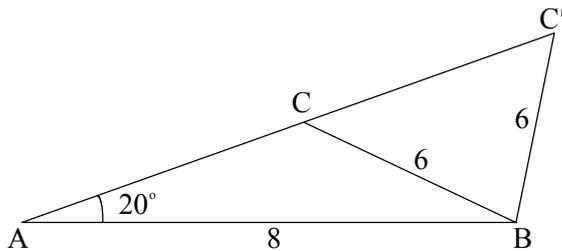
(M1)

$$\text{Area } \triangle ABC = 2.98 \text{ (cm}^2\text{) (accept 2.97)}$$

(A1)

(C6)

METHOD 2



Let $AC = x$

By the cosine rule $6^2 = 8^2 + x^2 - (2)(8)(x)\cos 20^\circ$

(M1)(A1)

$$\Rightarrow 0 = x^2 - 15.035x + 28$$

$$x = \frac{15.035 - \sqrt{(15.035)^2 - 112}}{2}$$

(A1)

$$= 2.178$$

(A1)

$$\text{Area} = \frac{1}{2}AB \times AC \sin(20^\circ) = \frac{1}{2}(8)(2.178) \sin 20^\circ$$

(M1)

$$= 2.98 \text{ (cm}^2\text{)}$$

(A1)

(C6)

[6 marks]

13. Using integration by parts $u = \theta$ $v = \sin \theta$ (M1)

$du = d\theta$ $dv = \cos \theta d\theta$

$\Rightarrow \int \theta \cos \theta d\theta = \theta \sin \theta - \int \sin \theta d\theta$ (M1)(A1)

$\Rightarrow \int \theta \cos \theta d\theta = \theta \sin \theta + \cos \theta$ (A1)

Therefore, $\Rightarrow \int (\theta \cos \theta - \theta) d\theta = \theta \sin \theta + \cos \theta - \frac{\theta^2}{2} + c$ (A2) (C6)

Note: Award (C5) for $\theta \sin \theta + \cos \theta - \frac{\theta^2}{2}$, i.e. penalize omission of + c by [1 mark].

[6 marks]

14. METHOD 1

$y = xe^x$

$\frac{dy}{dx} = xe^x + e^x$ (M1)(A1)

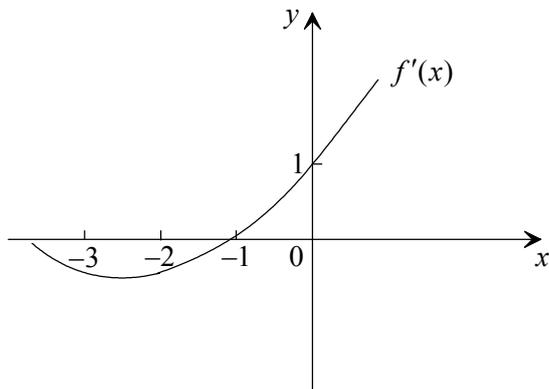
$\frac{d^2y}{dx^2} = xe^x + 2e^x$ (M1)(A1)

$= e^x(x + 2)$ (A1)

Therefore the x-coordinate of the point of inflexion is $x = -2$ (A1) (C6)

METHOD 2

Sketching $y = f'(x)$



(G4)

$f'(x)$ has a minimum when $x = -2$ (G1)

Thus, $f(x)$ has point of inflexion when $x = -2$ (G1)

[6 marks]

15. Let m be the median.

Then $\int_0^m \frac{1}{4}x(4-x^2) dx = 0.5$. (M1)

$\Rightarrow \int_0^m (4x - x^3) dx = 2$ (A1)

$\Rightarrow \left[2x^2 - \frac{1}{4}x^4 \right]_0^m = 2$ (M1)

$\Rightarrow 2m^2 - \frac{1}{4}m^4 = 2$

$\Rightarrow m^4 - 8m^2 + 8 = 0$ (A1)

$m = 1.08$ (G2)

OR

$m^2 = \frac{8 \pm \sqrt{64 - 32}}{2} = \frac{8 \pm \sqrt{32}}{2} = 4 \pm \sqrt{8} \left(4 \pm 2\sqrt{2} \right)$ (M1)

$\Rightarrow m = \sqrt{4 - \sqrt{8}} \left(\sqrt{4 - 2\sqrt{2}} \right)$ (A1) (C6)

Note: Award (C5) if other solutions to the equation $m^4 - 8m^2 + 8 = 0$ appear in the answer box.

[6 marks]

16. $\frac{dV}{dt} = 8(\text{cm}^3\text{s}^{-1}), V = \frac{4}{3}\pi r^3$

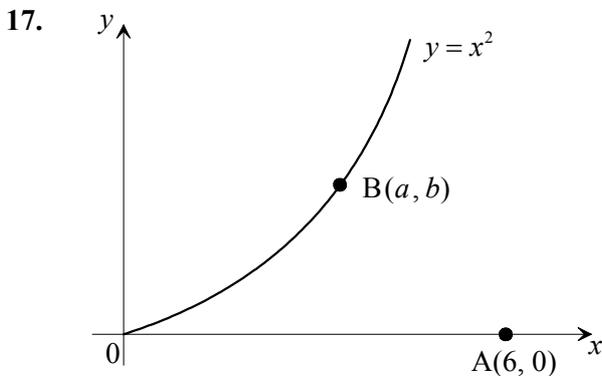
$\Rightarrow \frac{dV}{dr} = 4\pi r^2$ (M1)(A1)

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \left(\frac{dV}{dt} \right) \div \left(\frac{dV}{dr} \right)$ (M1)

When $r = 2, \frac{dr}{dt} = 8 \div (4\pi \times 2^2)$ (M1)(A1)

$= \frac{1}{2\pi}(\text{cm s}^{-1})$ (do not accept 0.159) (A1) (C6)

[6 marks]



$b = a^2$

$AB^2 = (a-6)^2 + a^4$ (M1)(A1)

Minimum value of $(x-6)^2 + x^4$ occurs at $x = 1.33$ (G3)

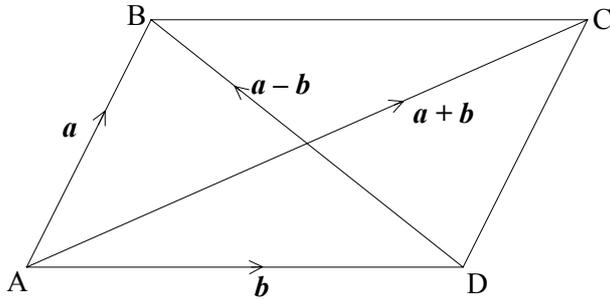
$\Rightarrow a = 1.33$ (G1) (C6)

[6 marks]

18. METHOD 1

Let $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ be diagonals of a parallelogram ABCD with sides AB and AD equal to \mathbf{a} and \mathbf{b} respectively.

(M1)



(M1)

If $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$ then the diagonals AC and BD are equal in length. Therefore ABCD is a rectangle and $\mathbf{a} \cdot \mathbf{b} = 0$.

(M1)(A1)

(R1)(A1)

(C6)

METHOD 2

$$|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$$

$$\Rightarrow |\mathbf{a} + \mathbf{b}|^2 = |\mathbf{a} - \mathbf{b}|^2$$

(M1)

$$\Rightarrow (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = (\mathbf{a} - \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b})$$

(M1)(A1)

$$\Rightarrow |\mathbf{a}|^2 + 2\mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2 = |\mathbf{a}|^2 - 2\mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2$$

(A1)

$$\Rightarrow 4\mathbf{a} \cdot \mathbf{b} = 0$$

(A1)

$$\Rightarrow \mathbf{a} \cdot \mathbf{b} = 0$$

(A1)

(C6)

[6 marks]

19. Matrix representing a reflection in $y = x\sqrt{3} = x \tan \frac{\pi}{3}$ is $\mathbf{M} = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ (A1)

Matrix representing an anticlockwise rotation about 0 through $\frac{\pi}{6}$ is $\mathbf{R} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ (A1)

$$\mathbf{RM} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \quad (M1)$$

$$= \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \quad (A1)$$

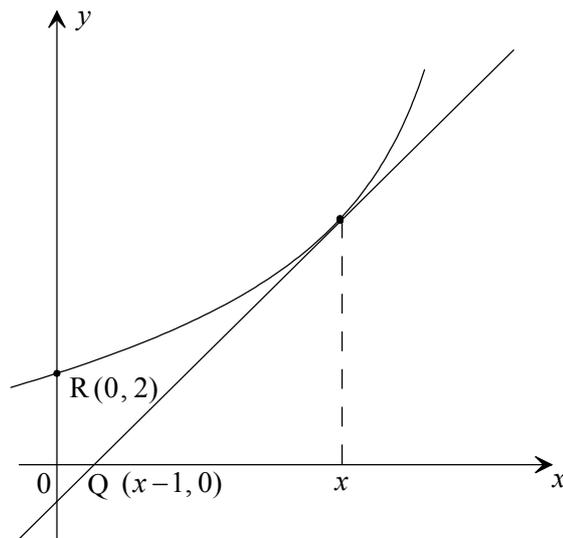
$$= \begin{pmatrix} \cos \frac{5\pi}{6} & \sin \frac{5\pi}{6} \\ \sin \frac{5\pi}{6} & -\cos \frac{5\pi}{6} \end{pmatrix}$$

This is the matrix representing a reflection in $y = x \tan \frac{5\pi}{12}$ (A1)(A1) (C6)

Notes: Candidates may use diagrams/geometry.
Award (M1)(A1) for a correct representation of \mathbf{M} ,
(M1)(A1) for a correct representation of \mathbf{R} ,
(A1)(A1) for the correct answer.

[6 marks]

20.



From the diagram,

$$\frac{dy}{dx} = \frac{y}{1}$$

(M1)(A1)

$$\Rightarrow \int \frac{dy}{y} = \int dx$$

(M1)

$$\Rightarrow \ln y = x + c$$

(A1)

$$\Rightarrow y = e^{x+c} = Ae^x.$$

(A1)

But $R(0, 2)$ lies on the curve and so $A = 2$.

(A1)

Thus $y = 2e^x$

(C6)

[6 marks]
